

Athemath Spring 2023 Admissions Quiz

Athemath Staff

Due January 21st, 2023

§1 Instructions

For all of the problems below, **proof-based solutions are encouraged**. We would like you to explain all of your steps, instead of just giving an answer. If you don't have experience with proofs, just try to explain your answer as much as you can. \LaTeX submissions and *neat*, dark handwriting submissions are both allowed.

We also encourage you to try the entire test. While later problems will generally be harder, they also play to different strengths and you may find one particularly easy.

Please do not use computer programs, Google, WolframAlpha, GeoGebra, etc. to help you find solutions. Additionally, please do not discuss this quiz with anyone else until after the application deadline has passed. **If you find the test difficult, that's because it's designed to be.** If you get stuck, take a walk, try a different problem, or try a strategy you dismissed at first. And remember that you don't have to solve all—or even a majority—of the problems to get in. Historically, the average admitted student solves around two problems.

Ask for clarifications by emailing Serena at serena.an@themath.org. Submit your completed solutions to the [application form](#) by **January 21st, 11:59PM Eastern**. As a reminder, only students of underrepresented genders can apply. Have fun!

§2 The Problems!

Problem 1

What is the expected value of

$$\frac{x_1}{1} + \frac{x_1 + x_2}{2} + \frac{x_1 + x_2 + x_3}{3} + \cdots + \frac{x_1 + x_2 + x_3 + \cdots + x_{100}}{100},$$

where $(x_1, x_2, x_3, \dots, x_{100})$ is a permutation of $(1, 2, 3, \dots, 100)$ chosen at random?

Problem 2

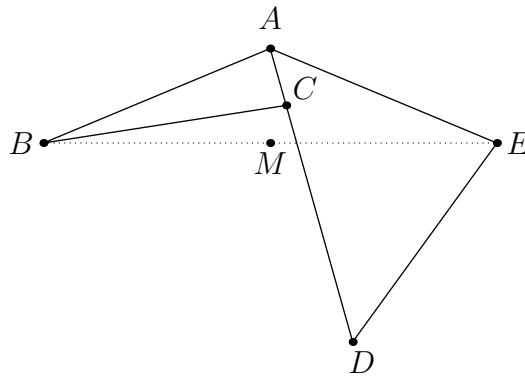
Given that there are exactly 10 positive integer solutions (x, y) to $20x + 23y = k$, find the largest possible value of k .

Problem 3

Ali has 2022 plants of weights $1, 2, \dots, 2022$ and 2022 pots that can support plants of weights up to $2, 3, \dots, 2023$. How many ways can she place each plant into a different pot while satisfying the weight restriction?

Problem 4

Pentagon $ABCDE$ has all sides of length 13, and C lies on \overline{AD} . Let M be the midpoint of BE . Given that $AM = 5$, find the area of triangle CMD .



Problem 5

Find all ordered pairs of integers (a, b) such that $a + b^2$ and $a^2 + b$ are both powers of 2.