Athemath Fall 2023 Admissions Quiz

Athemath Staff

Due September 2nd, 2023

§1 Instructions

For all of the problems below, **proof-based solutions are encouraged**. We would like you to explain all of your steps, instead of just giving an answer. If you don't have experience with proofs, just try to explain your answer as much as you can. LaTeX submissions and *neat*, dark handwriting submissions are both allowed.

We also encourage you to try the entire test. While later problems will generally be harder, they also play to different strengths and you may find one particularly easy.

Please do not use computer programs, Google, WolframAlpha, GeoGebra, etc. to help you find solutions. Additionally, please do not discuss this quiz with anyone else until after the application deadline has passed. **If you find the test difficult, that's because it's designed to be.** If you get stuck, take a walk, try a different problem, or try a strategy you dismissed at first. And remember that you don't have to solve all of the problems to get in. Historically, the average admitted student solves around two or three problems.

Ask for clarifications by emailing Serena at serena.an@athemath.org. Submit your completed solutions to the application form by **September 2nd, 11:59PM Eastern**. As a reminder, only students of underrepresented genders can apply. Have fun!

§ 2 The Problems!

Problem 1

Each edge and diagonal of a regular hexagon (15 in total) is colored red or black such that the hexagon appears unchanged when rotated by 60 degrees clockwise about its center. How many such colorings are possible?

Problem 2

Suppose a and b are positive integers such that $a^2 + b^2$ is even and $a^3 + b^3$ is a multiple of 3. What is the largest positive integer that must divide $a^5 + b^5$?

Problem 3

The common chord and a common tangent of two intersecting circles are both one unit long. If one circle has twice the radius of the other, determine the distance between their centers.

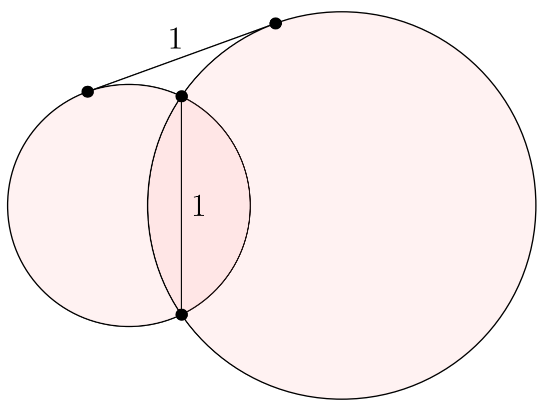

Figure not to scale.

Problem 4

Suppose x is a *real* number such that $x^5 - (x-1)^5 = \frac{1}{5}$. Determine all possible values of $x^4 - (x-1)^4$.

Problem 5

A 100×100 checkerboard is tiled with dominoes (2×1 or 1×2 rectangles), with each square of the checkerboard covered by exactly one domino. Prove that there must exist two dominoes that together form a 2×2 square.