Athemath Spring 2026 Diagnostic

Athemath Staff

Due January 25, 2026

§1 Instructions

This is not a quiz! It is simply for us to get a sense of your current math level, so that we can determine whether or not you will benefit from Athemath classes, and if so, which Athemath classes would be appropriate for you. You are not required to solve all or even most of the problems, although we do encourage you to try your best on all of them! While later problems will generally be harder, they also play to different strengths and you may find one particularly easy.

For all problems, **proof-based solutions are encouraged**. We would like you to explain all of your steps instead of just giving an answer, and even if you can't solve a problem, we'd love to see your progress! If you don't have experience with proofs, just try to explain your answer as much as you can. LaTeX submissions and *neat*, dark handwriting submissions are both allowed.

Please **do not use** computer programs, large language models (e.g. ChatGPT), Google, WolframAlpha, etc. to help you find solutions (however, GeoGebra is allowed). Additionally, please do not discuss this quiz with anyone else until after the application deadline has passed. If you find the test difficult, that's because it's designed to be! If you get stuck, take a walk, try a different problem, or try a strategy you dismissed at first. And remember that you don't have to solve all of the problems to get in. Historically, the average admitted student solved around two problems, though the difficulty varies over the years.

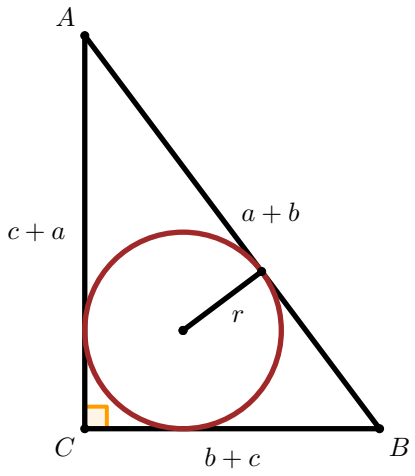
Ask for clarifications by emailing contact@athemath.org. Submit your completed solutions to the application form by **January 25th**, Anywhere on Earth. However, the sooner you apply, the sooner you might hear back from us! As a reminder, only students of underrepresented genders can apply. Have fun!

Please justify your answers and show your work!

§ 2 The Problems!

Problem 1

Let ABC be a right triangle with $\angle C = 90^\circ$ and side lengths given by $AB = a + b$, $BC = b + c$, $CA = c + a$. A circle with radius r is drawn inside the triangle touching all three sides.



(a) Find c and r if $a = 3$ and $b = 2$.

(b) In general, find a relation between r and c, and try to prove it.

Problem 2

Note: Visit https://athemath.org/misc/battle for an interactive simulation of this problem!

A character in a game is fighting n enemies. The ith enemy has health a_i and attack b_i. On the player's turn, they decrease the health of any selected enemy by 1. The enemy dies if it reaches 0 health. On the enemies' turn, each enemy that is still alive attacks the player, with the ith enemy doing b_i damage.

(a) If $n = 2$, $(a_1, b_1) = (2, 5)$, and $(a_2, b_2) = (3, 6)$, what's the optimal strategy to minimize damage?

Hint: You should find that the optimal strategy includes taking down the enemies one at a time. Try to show this!

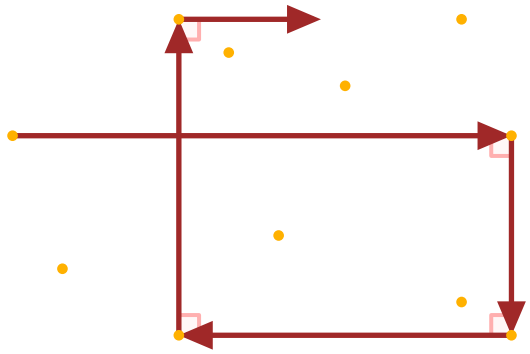
(b) In general, for $n = 2$, what's the optimal strategy to minimize damage?

(c) Find the strategy for any given n and list of (a_i, b_i) to minimize damage.

Problem 3

Arrange a finite set S of points on the plane and let a point of light L to the left of all points in S start moving right. Whenever L hits a point in S, it turns 90° clockwise and continues moving straight.

Prove that L will eventually be moving in some direction indefinitely.



Problem 4

Find the number of integers n between 1 and 2026 inclusive such that $\gcd(2^n + 1, 2^n + 2^i) = 1$ for all $1 \leq i \leq n$.

Problem 5

For a permutation $a_1, a_2, \ldots, a_n$ of the sequence $1, 2, \ldots, n$, let

$$S = \min(a_1, a_2) + \min(a_2, a_3) + \cdots + \min(a_n, a_1).$$

(a) Find the maximum value of S.

(b) Find the number of sequences such that this maximum value is attained.

§ 3 Special Thanks

A special thanks to Evan Chen, Jiahe Liu, Jiya Dani, Patrick Suwanich, Arjun Suresh, and Christopher Bao for the problem proposals, in order of problem proposed!