Athemath Fall 2026 Diagnostic

Athemath Staff

Due August 31st, 2026

§1 Instructions

This is not a quiz! It is simply for us to get a sense of your current math level so that we can determine whether or not you will benefit from Athemath classes, and if so, which Athemath classes would be appropriate for you. You are not required to solve all or even most of the problems, although we do encourage you to try your best on all of them! While later problems will generally be harder, they also play to different strengths and you may find one particularly easy.

For all problems, **proof-based solutions are encouraged**. We would like you to explain all of your steps instead of just giving an answer. If you don't have experience with proofs, just try to explain your answer as much as you can. LaTeX submissions and *neat*, dark handwriting submissions are both allowed.

Please do not use computer programs, large language models (e.g. ChatGPT), Google, WolframAlpha, etc. to help you find solutions (however, GeoGebra is allowed). Additionally, please do not discuss this quiz with anyone else until after the application deadline has passed. If you find the test difficult, that's because it's designed to be! If you get stuck, take a walk, try a different problem, or try a strategy you dismissed at first. And remember that you don't have to solve all of the problems to get in. Historically, the average admitted student solved around two problems, though the difficulty varies over the years.

Additionally, we request that you **submit your scratch work** for each of the problems; there'll be a spot in the application form to upload it. While this isn't mandatory and we won't look at your scratch work unless necessary, it'll be helpful for us to see your thought process as you work through problems.

Ask for clarifications by emailing contact@athemath.org. Submit your completed solutions to the application form by the end of the day on **August 31st, Anywhere On Earth**. As a reminder, only students of underrepresented genders can apply. Have fun!

Please justify your answers and show your work!

§ 2 The Problems!

Problem 1

Dora has multiple piles of stones in front of her, arranged in order from least to most stones. No two piles have the same number of stones. Swiper has the same piles of stones in front of him. They both wish to have the number of stones in their piles be a list of consecutive integers. In one move, Dora may add a stone to any of her piles, and Swiper may swipe (i.e. remove) a stone from any of his piles, assuming there'll be at least one stone remaining. Let D and S denote the minimum number of moves Dora and Swiper each need to make their piles consist of consecutive numbers of stones. Say an original configuration is *curious* if $D = S$.

For example, in the configuration $\{2, 4, 5\}$, Dora adds one stone to 2 to obtain $\{3, 4, 5\}$, so $D = 1$. Swiper removes one stone from 4 and one stone from 5 to obtain $\{2, 3, 4\}$, so $S = 2$. Therefore, this configuration isn't curious.

(a) Show that the configuration $\{4, 6, 7, 9\}$ is curious and compute the value of $D = S$.

(b) Find all values of x such that adding a pile of x stones to the configuration $\{1, 5, 15, 23, 25\}$ makes it curious.

Hint: One way you can do this is by finding a nice way to describe all curious configurations!

Problem 2

Lanie has a collection of 12 coins, some of which may be counterfeit. She digs 12 small holes in a circle and buries one coin in each hole. Selena has a device which, for a fixed positive integer $m \leq 12$, can determine the number of counterfeit coins in any m consecutive holes around the circle. For which values of m is Selena guaranteed to be able to determine exactly which holes contain counterfeit coins, regardless of the original arrangement?

Problem 3

Greta and Rachel are trying to find Vivian in a two-dimensional grid. The grid has dimensions 10×10, with each position being one of the unit cells. At the start of the game, everyone occupies a distinct cell, and all their locations are visible throughout the game. On each move, Vivian starts by moving to an adjacent cell. Shortly after, Greta and Rachel each simultaneously move to a cell adjacent to their previous positions. Greta and Rachel win if either one of them occupies the same cell as Vivian at some point.

(a) Prove that without Rachel, there exists a starting configuration where Greta can't find Vivian.

(b) Prove that Greta and Rachel can find Vivian together, regardless of where everyone starts.

(c) If the game is played in a cube with dimensions $10 \times 10 \times 10$ instead, can Greta and Rachel still win, regardless of their starting positions?

Problem 4

Let n be a positive integer. Catherine writes the numbers 1 through n^2 in an $n \times n$ grid G, in order from left to right and top to bottom. She then modifies the grid by replacing every integer k with $\gcd(k, n^2 + 1)$. Prove that this grid is unchanged by a $90°$ clockwise rotation.

Below is the original grid for $n = 3$.

1	2	3
4	5	6
7	8	9

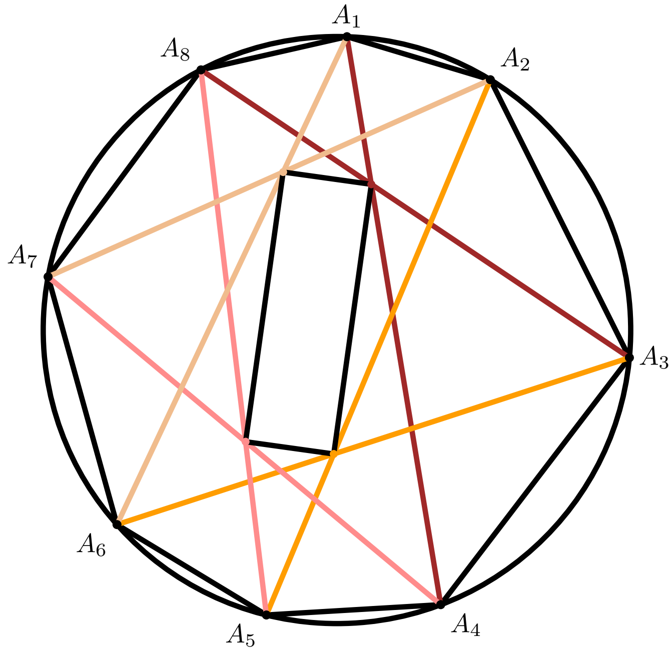
Below is the grid with the replacements.

1	2	1
2	5	2
1	2	1

Problem 5

Points $A_1, \ldots, A_8$ lie on a circle in that order such that $A_iA_{i-1} = A_iA_{i+1}$ for $i \in \{1, 3, 5, 7\}$. Prove that the intersections of $A_{j-2}A_{j+1}$ and $A_{j-1}A_{j+2}$ for $j \in \{2, 4, 6, 8\}$ form a rectangle.

Note that indices are taken mod 8.



§ 3 Special Thanks

A special thanks to Arjun Suresh (problems 1, 2, 3), Tarun Rapaka (problem 4), and Benjamin Fu (problem 5) for the problem proposals!